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Void Fractions in Two-phase Steam-water Flow

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The pressure-drop characteristics associated with one liquid and one gaseous phase flowing concurrently in a pipe or tube have yet to be understood. The operation of evaporators, boilers, and condensers has long stimulated interest in the pressure drop of steam-water mixtures, and more recently this specialized case of one-component, two-phase flow has received even greater attention from the applications in cooling nuclear reactors. The two-phase-flow problems have not been amenable to thorough theoretical analyses, and therefore empirical and semiempirical correlations have attained unusual prominence in practical applications. The present investigation employs a new research tool for the study of two-phase-flow structure.

A variety of geometric flow patterns is possible. Bergelin, Alves, and others have classified these patterns according to visual appearance; whereas the Martinelli classifications were based upon whether the flow in each phase was termed *viscous* or *turbulent*. The distinction between viscous and turbulent flow in either phase is rather arbitrary, and if the Reynolds number for one phase, calculated on the basis of the total tube diameter, is greater than 2,000, the flow in the phase is called *turbulent*. This investigation is confined to the study of annular flow, in which most of the liquid is found in an annular ring surrounding the central vapor core and the flow in each phase is turbulent.

Boiling or flashing occurs when superheated water rises in an insulated vertical tube at atmospheric pressure. For a separated two-phase flow geometry, the mean linear steam velocity may exceed that of the water. The fraction of the tube occupied by the steam (void fraction) at a given cross section cannot be obtained directly from a determination of the thermodynamic quality. Void fractions, however, must be known for the estimation of the pressure drops due to head and momentum changes.

Void fractions and pressure drops for steam-water flows were measured in an 0.872-in. I.D. vertical tube at atmospheric pressure over a quality range of 0 to 4%. The test section was the hot leg of a natural-circulation loop, and the inlet liquid flow rate ranged from 1 to 3 ft./sec. A new technique for measuring void fractions was used, and the method utilizes the difference between the gamma-ray absorption coefficients of water and steam.

A comprehensive survey of two-phase frictional pressure drops has been prepared by Isbin, Mosher, and Moen (*9*), and an additional brief survey is given by Marchaterre (*13*). The experimental data of this investigation are compared with the predictions of the homogeneous and Martinelli models.

The treatment of the vapor-liquid mixture as a homogeneous fluid is called the homogeneous, fog, or Woods model (*12*). The two phases are assumed to be in equilibrium, average specific volume and viscosity properties are used, and the mean linear velocities of the vapor and the liquid are assumed to be equal. For example, the total pressure drop for a steam-water mixture flowing vertically upward in a channel of uniform cross section is approximated by the following equation for steady-state conditions:

$$p_n - p_{n+1} = \left(\frac{z_{n+1} - z_n}{\bar{v}}\right)\frac{g}{g_c} + \frac{G^2}{g_c}(v_{n+1} - v_n) + \frac{\bar{f}G^2\bar{v}(z_{n+1} - z_n)}{2g_cD} \quad (1)$$

static pressure drop	head pressure drop	momentum pressure drop	frictional pressure drop

where

$$v_n = q_n v_{sn} + (1 - q_n)v_{wn} \quad \text{and} \quad \frac{1}{\mu_n} = \frac{q_n}{\mu_s} + \frac{1 - q_n}{\mu_w}$$

Fanning friction factor $\bar{f}$ is evaluated at an average Reynolds number average property between stations, for example, $\bar{v} = (v_n + v_{n+1})/2$. $\bar{f}$ and $\bar{q}$ also are average values.

(The use of an arithmetic mean was sufficiently accurate for the conditions selected.)

The Martinelli method is characterized by two basic postulates: (1) for the case of steady, two-phase flow involving no radial pressure gradients, the frictional pressure drop is assumed to be the same in both the liquid and gas phases, and (2) at any instant the sum of the volumes occupied by the liquid and gas must be equal to the total volume of the tube. In terms of the fractional cross-sectional areas occupied by each phase, $R_l + R_g = 1$.

Martinelli and Lockhart (*11*) presented a two-phase frictional pressure-drop correlation in the form of related dimensionless parameters. The param-

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eters involve the ratio of the two-phase frictional pressure drop to single-phase frictional pressure drop and the ratio of the single-phase frictional pressure drop of the gas to the corresponding value for the liquid. Parameters similar to those of Martinelli were derived by Levy (*10*), who carried out a theoretical analysis for annular flow, and by Gazley (*7*), who also developed a semiempirical method. In general, the Martinelli correlation was developed from extensive studies of two-component, two-phase flow with no mass transfer between phases. The adaptation of the correlation to the flow of flashing steam-water mixtures and the introduction of an empirical pressure dependency were carried out by Martinelli and Nelson (*14*).

The total pressure drop for a steam-water mixture flowing vertically upward in a channel of uniform cross section is approximated by the following equation by use of the Martinelli correlation:

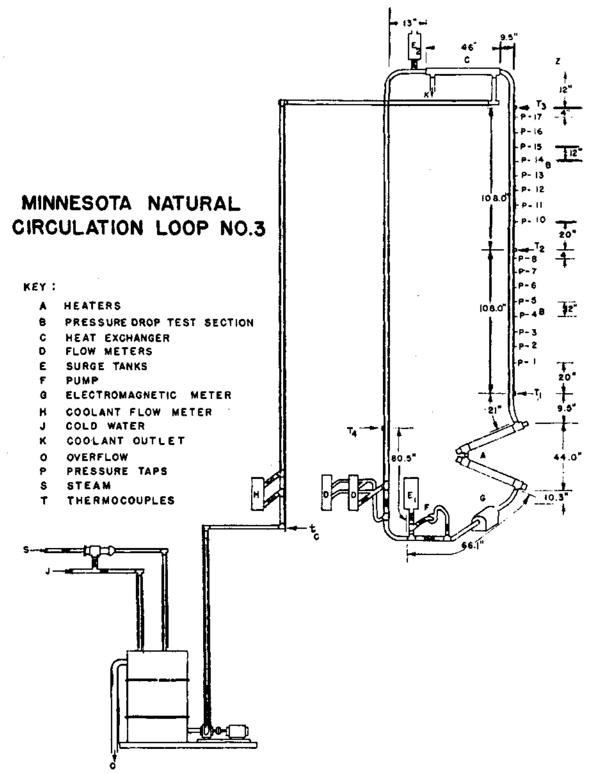

Fig. 1. The natural-circulation loop.

$$\underbrace{p_n - p_{n+1}}_{\text{static pressure drop}} = \underbrace{\left(\frac{z_{n+1} - z_n}{\bar{v}}\right)\frac{g}{g_c}}_{\text{head pressure drop}}$$

$$+ \underbrace{\frac{G^2}{g_c}\left\{\left[\frac{v_w(1-q)^2}{R_l}\right]_{n+1} + \left[\frac{v_s q^2}{R_g}\right]_{n+1} - \left[\frac{v_w(1-q)^2}{R_l}\right]_n - \left[\frac{v_s q^2}{R_g}\right]_n\right\}}_{\text{momentum pressure drop}}$$

$$+ \underbrace{\phi^2_{ltt}\left[\frac{\bar{f}G^2(1-\bar{q})^2}{2g_cD}v_w\right][(z_{n+1} - z_n)]}_{\text{frictional pressure drop}} \qquad (2)$$

where

$$\frac{2}{\bar{v}} = \left(\frac{R_g}{v_s} + \frac{R_l}{v_l}\right)_n + \left(\frac{R_g}{v_s} + \frac{R_l}{v_l}\right)_{n+1}$$

$\bar{f}$ is the Fanning friction factor evaluated for the average liquid

Reynolds number over interval n to $n+1$,

$$\overline{Re} = \frac{(1-\bar{q})GD}{\mu_{sat\ liquid}}$$

$$\bar{q} = \tfrac{1}{2}(q_n + q_{n+1})$$

ϕ^2_{ltt}, the Martinelli two-phase frictional pressure-drop factor, is evaluated for the

TABLE 1

HOMOGENEOUS, MARTINELLI, AND EXPERIMENTAL PRESSURE DROPS

Run	q', %	W, lb./sec.	Homogeneous model, lb./(sq. ft.)(ft.) $(\Delta P/\Delta L)_H$	$(\Delta P/\Delta L)_M$	$(\Delta P/\Delta L)_{TPF}$	$(\Delta P/\Delta L)_{TP}$	Martinelli correlation, lb./(sq. ft.)(ft.) $(\Delta P/\Delta L)_H$*	$(\Delta P/\Delta L)_M$*	$(\Delta P/\Delta L)_{TPF}$	ϕ_{ltt}	Experimental, lb./(sq. ft.)(ft.) $(\Delta P/\Delta L)_{TP}$
22	0.75	0.707	5.2	19.4	22.0	46.6	16.2	4.2	22.1	3.34	42.5
23	1.26	0.521	3.1	11.8	22.2	37.1	12.7	1.3	18.0	3.91	32.0
24	0.78	0.342	4.6	3.7	6.9	15.2	15.8	0.4	7.8	3.78	24.0
25	1.29	0.704	3.0	37.2	38.8	79.0	12.6	4.1	36.2	4.18	52.9
26	1.17	0.520	3.1	10.6	21.2	34.9	13.2	1.4	16.0	3.59	30.6
27	1.03	0.347	3.4	4.4	8.0	15.8	14.0	0.4	7.7	3.73	22.1
28	0.39	0.706	10.2	18.3	11.2	39.7	21.2	2.4	18.1	3.03	41.7
29	0.48	0.523	7.9	8.8	8.5	25.2	19.5	2.0	11.2	3.08	32.7
30	0.44	0.343	8.1	2.8	3.9	14.8	20.3	1.2	5.5	3.19	27.0
31	0.39	0.358	9.0	3.7	3.8	16.5	21.2	1.3	5.6	3.08	28.1
32	0.48	0.527	7.9	8.3	8.4	24.6	19.5	2.0	11.5	3.20	33.0
33	0.36	0.670	10.8	12.7	9.8	33.3	22.1	4.7	15.1	2.90	41.9
34	0.40	0.750	10.3	20.6	12.2	43.1	20.9	2.9	18.4	2.90	42.2
35	2.44	0.478	1.9	8.6	27.4	37.9	9.2	8.4	18.1	4.49	35.7
36	4.26	0.352	1.0	7.5	41.5	50.0	6.2	3.6	21.5	6.37	31.3
38	0.86	0.534	4.7	7.8	14.7	27.2	15.2	1.0	14.4	3.48	30.6
39	1.29	0.346	3.1	2.6	10.0	15.7	12.6	1.0	9.2	4.10	22.8
40	1.24	0.666	3.0	24.8	33.5	61.3	12.8	2.7	30.5	4.17	46.0
41	1.28	0.532	2.9	10.2	23.7	36.8	12.6	1.3	18.2	3.92	32.1
42	1.69	0.338	2.2	2.9	13.7	18.8	10.2	5.1	8.8	4.11	24.1
43	1.35	0.711	2.8	26.5	40.7	70.0	12.3	2.6	37.1	4.33	52.0
44	1.18	0.573	3.1	10.0	25.0	38.1	13.1	1.1	24.1	3.54	38.3
45	4.18	0.352	0.9	16.2	32.8	49.9	6.3	1.4	25.9	6.93	33.6

*Computed from experimental R_l.

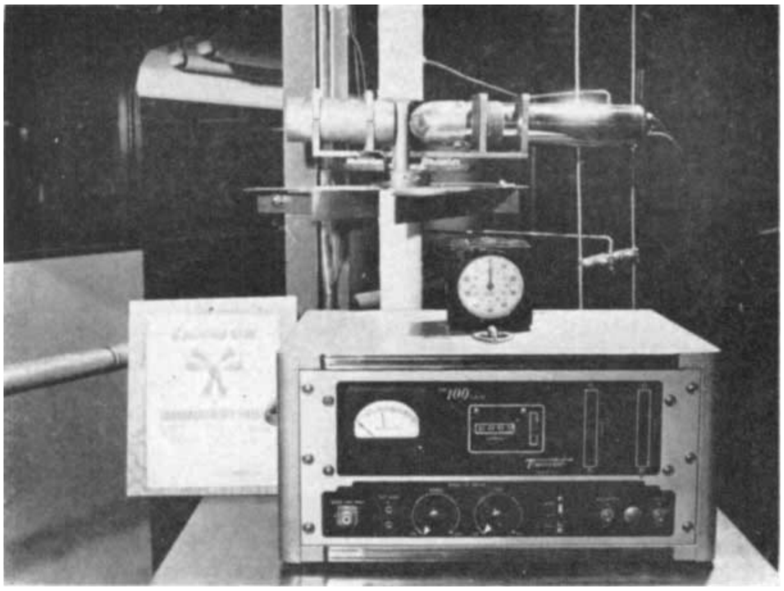

Fig. 2. Gamma absorption equipment.

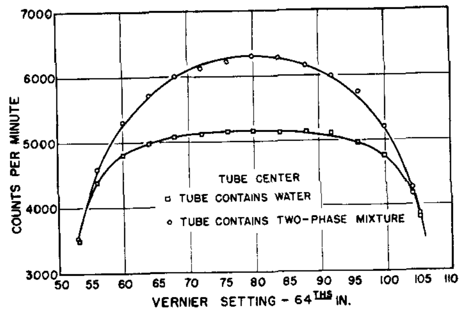

Fig. 4. Counting rates for typical run (run 28).

flow conditions averaged over interval n to $n + 1$.

One assumes that the use of a mean velocity for each phase will describe adequately the momentum contributions.

The Martinelli correlation includes the evaluation of the liquid holdup based upon experimental measurements, most of which were made by trapping the liquid in the flow channel (*11*). In recent years radioactive methods have been employed for the measurement of liquid holdup. Studies at the University of Minnesota and elsewhere were carried out independently. Tomlinson (*17*) studied the application of a gamma-ray absorption technique, and Eddy (*6*) designed and constructed the gamma-absorption equipment used for studying steam-water flows. Dengler (*5*) added a radioactive tracer to water and calculated the liquid holdup from measurements of the radiation intensity. A thulium gamma source has been used by the investigators at the Argonne National Laboratory, and the results are presented by Marchaterre (*13*). Zmola and Bailey (*19*) report the use of iridium 192 for measurement of densities of a boiling liquid. Schwarz (*15*) measured liquid holdups by a gamma-absorption method using iridium 192. Measurements were made with horizontal and vertical steam-water flows at pressures as high as 80 atm. A beta-ray–absorption technique (with strontium 90 employed as a source) has been used by Anson, Belin, and Horlor (*3*) to measure the density of steam-water mixtures in the throat of a critical-flow nozzle. The use of X-ray techniques is not included in this survey; an example of an application to gas-fluidized-solid systems may be found in a study by Grohse (*8*).

RADIATION-ABSORPTION METHOD

The attenuation of monoergic parallel gamma rays passing through a thin homogeneous absorbing medium of uniform thickness is given by the equation

$$\frac{I}{I^*} = e^{-\beta x} \qquad (3)$$

where I/I^* is the fraction of the incident radiation penetrating a distance x in the absorber. The linear absorption coefficient β is dependent upon the nature of the absorbing material, absorber density, and radiation energy. The coefficient was determined experimentally so as to take into account deviations from Equation (3) arising from scattering and the gamma energy spectrum.

Application of Equation (3) is made for the flow of fluids in a pipe or tube. For a beam of gamma rays of finite dimensions passing through a tube diameter or chord, for example, the measured values of the radiation are I_m for a beam through a tube diameter D with the tube empty and I_w for a

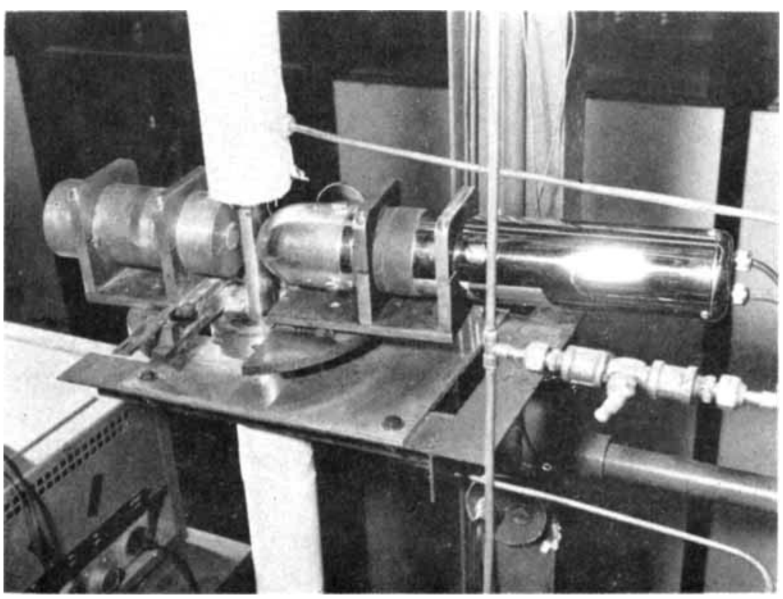

Fig. 3. Detailed view of carriage.

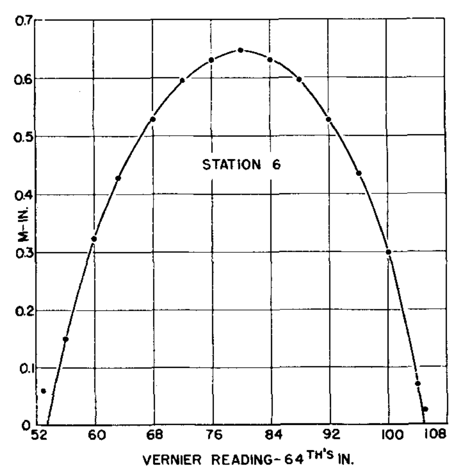

Fig. 5. Steam chordal lengths vs. vernier setting for graphical integration (run 28).

beam through a tube diameter with the tube full of water. The approximate relation between I_m and I_w is

$$I_w = I_m e^{-\beta_w D} \qquad (4)$$

If I_{TP} represents the measured value of the radiation passing through a chordal length c in a steam-water mixture, then

$$I_{TP} = I_w e^{m(\beta_w - \beta_s)} \qquad (5)$$

The effective chordal length of steam is m, the chordal length of water is l, and $m + l = c$. At low pressures the absorption coefficient for the steam β_s is very much smaller than the liquid-water value β_w and can be neglected.

The chordal length of steam m can be determined at any position in the tube by radiation measurements with the tube containing the two-phase mixture, preceded or followed by a measurement with the tube full of water at the same temperature (source decay neglected). A procedure of this type minimizes errors due to variations in the tube-wall thickness and the effect of scattering at the different measuring positions. The experimental value of β_w is calculated from Equation (4) and the water measurement is made at the appropriate temperature. This method neglects the temperature effect upon the density of the tube material. The gamma beam has a finite diameter, and thus the measured m corresponds to a mean position. All calculations were made on the assumption that the value for m actually corresponds to the chordal length of steam existing at the center of the gamma beam. Some of the consequences of this assumption are discussed in reference *16*.

The values for m are plotted vs. the position (center of the beam) at which each measurement is made, and a graphical integration of such a plot yields the fraction of the cross-sectional tube area occupied by the steam. This quantity may also be taken as the volumetric fraction occupied by the steam R_g at the given cross section. The value R_g is termed the *void fraction*, and $1 - R_g$, or R_l, is called the *liquid holdup*.

For symmetrical flow with the water in an annular ring, an accurate measurement of the position at which m becomes zero indicates the thickness of the water ring. Some insight into the actual liquid distribution might be gained if a radial distribution could be found which accurately accounts for the remaining water in the core. A correlative attempt of this type was not successful. Further discussions may be found in reference *16*.

EXPERIMENTAL

Natural-circulation Loop

The two-phase–flow test section used in these studies was an 0.872-in. I.D. vertical tube which was the hot leg of a natural-circulation loop. The loop has been de-

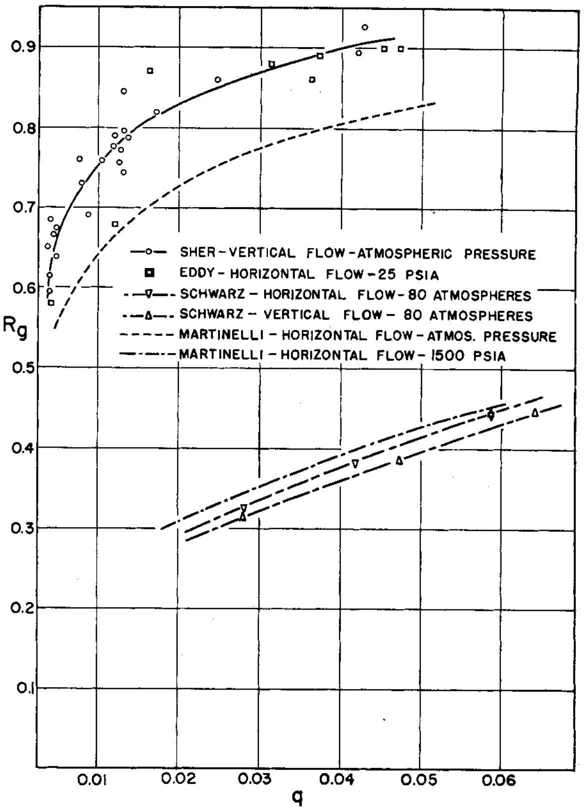

Fig. 6. Comparison of void measurements.

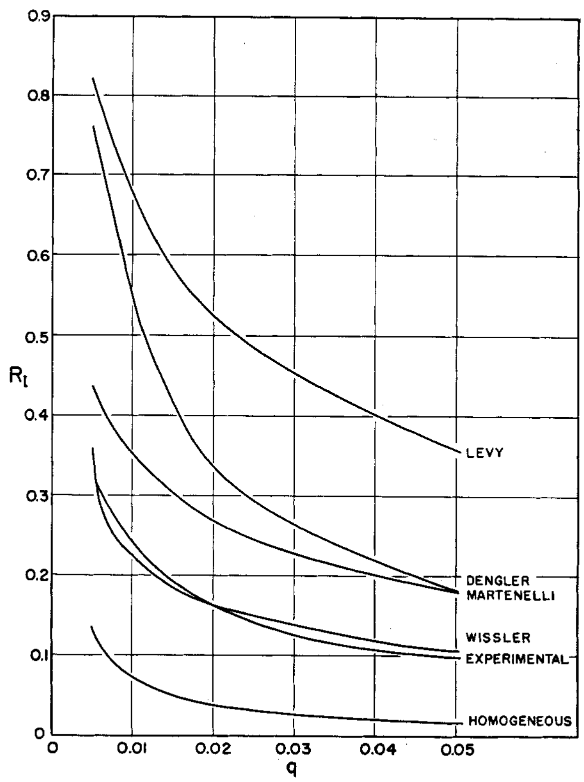

Fig. 7. Experimental liquid hold-up curve compared with predictions at atmospheric pressure.

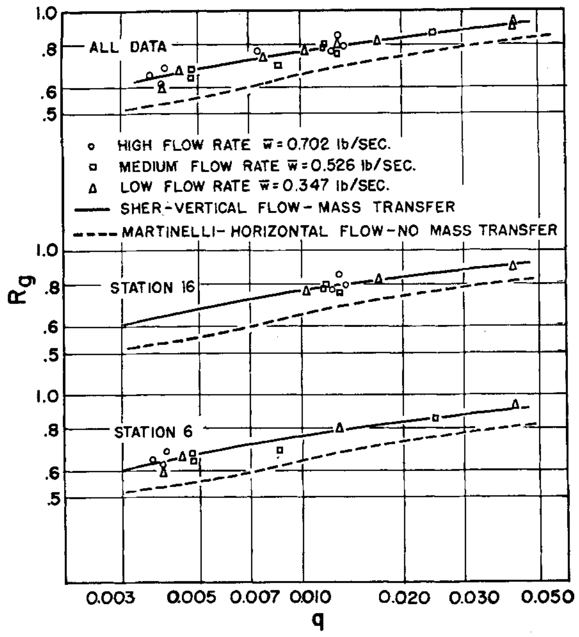

Fig. 8. Void data.

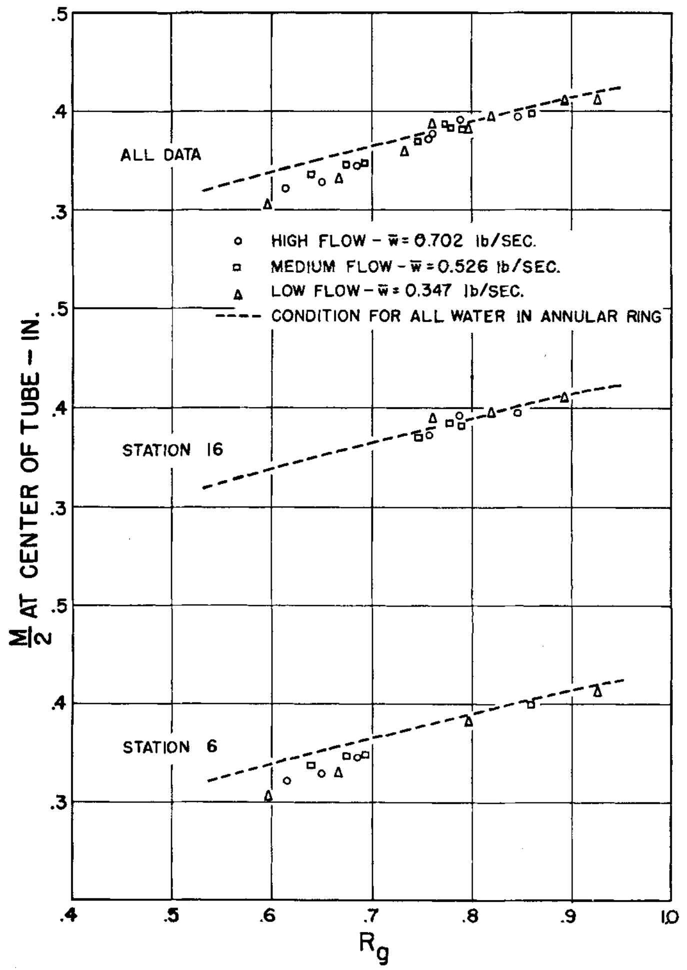

Fig. 9. Comparison of experimental chordal steam length at tube center with annular-flow-pattern calculation.

scribed in detail in references *1* and *18*. Figure 1 illustrates the main components of the loop.

The electromagnetic flow meter was calibrated each day that a series of runs was to be made. The best flow control during two-phase operation was achieved by restricting the natural circulation. Small flow fluctuations were observed, and these (±0.008 lb./sec. *max.*) increased with flow rate. Single-phase temperature measurements were uniform, but two-phase temperature measurements fluctuated ±0.16°F. The power input to the loop was measured by two recording watt meters, and the heat removal in the cooler was controlled by the coolant temperature and flow rate.

The surge tank at the top of the loop was open to atmospheric pressure, and the static pressures at the measuring stations varied from atmospheric to slightly higher values. Three flow rates were selected to cover the range of operation of the natural-circulation loop, the average values being 0.702, 0.526, and 0.347 lb./sec.

The maximum quality was just over 4%. The experimental errors in the calculated qualities ranged from about ±15% at a quality of 0.0025 to less than 1.0% at the maximum qualities. Thermodynamic equilibrium was assumed for these calculations; however, if superheated water were present, an additional error would be introduced. Plots of the raw pressure-drop data vs. quality revealed that the curves for similar flow rates had the same shape but were displaced from each other by as much as 0.1% quality. The average displacement was less than 0.05% quality, and this displacement corresponded to a water-temperature error of 0.5°F. The experimental errors did not account for this discrepancy, and therefore the conclusion was reached that the water was flashing from a superheated state. The calculated qualities (based upon thermodynamic equilibrium) were corrected so that the quality would be zero at the position where the pressure drop per unit length started to decrease. These corrections are given in reference *18*. The qualities reported in this paper and used in the pressure-drop calculations were the corrected values.

Method for Void Measurements

Selenium 75, with a half-life of 127 days, was used as the gamma source. About 10 mcurie. of selenium was contained in a 4-in.-diam. lead cylinder fitted with a 1/16-in.-diam. aperture. The source and a scintillation counter (model DS-1, Nuclear Instrument and Chemical Corporation) were mounted on a carriage which could be rotated around the tube wall and could be moved from one side of the tube to the other, thus traversing all chord positions. The carriage, carriage support, and tube mounting, source, scintillation tube, and scaler (model SC-7, Tracerlab 100) are shown in Figures 2 and 3.

The relationship between the readings of the vernier on the carriage and the actual position of the gamma beam in the tube was established for each tube mounting by determining the vernier readings for minimum gamma counting. These readings corresponded to the location of the inner tube wall. Traverses were made across the tube cross section by moving the carriage from one side of the tube to the other and back. Measuring positions for the return traverse were the same as for the forward traverse, and the counting rates for each position were averaged. The maximum possible error in locating the center of the tube was ±0.0039 in. Counting rates were about 4,000 counts/min. and greater, and at least 30,000 counts were obtained for each reading. (At 30,000 counts the probable statistical error is 1%.) The experimental value of β_w for each series of runs could be reproduced accurately to three figures, and a typical value was 0.310 in.$^{-1}$ The cross-sectional area of the tube, when full of water, could be determined within an error of less than 2%.

A typical two-phase run is illustrated in Figure 4, and a plot of the chordal steam lengths vs. the vernier readings is given in Figure 5. In general, the liquid distribution was found to be symmetrical about the axis of the vertical tube, and therefore it was not necessary to rotate the carriage about the tube to check the chordal measurements. The degree of reproducibility attainable is illustrated by runs 29 and 32: flow rate in lb./sec., 0.523 and 0.527; $(\Delta P/\Delta L)_T$ in lb./(sq. ft.)(ft.), 32.7 and 33.0; quality, 0.00475 and 0.00475; and void fraction R_g, 0.674 and 0.638. The station number refers to the vertical position of the gamma equipment and has the same value as the number of the pressure tap just below the carriage.

Inasmuch as the gamma beam was approximately 1/16 in. in diameter, counting rates taken for two-phase flow at positions closer than 1/16 in. from the inner edge of the tube could not be used. A quantitative correction for thickness of the gamma beam would involve the description of the annular flow pattern. (A more complete discussion is given in reference *16*.)

Although the steady state operation of the loop was carefully controlled during each run, extremely small variations were practically unavoidable. Since the values of R_g are quite sensitive to quality over a considerable range, the gamma-absorption data were examined for any deviations due to the rather long time-averaged measurements involved. The procedure resulted in readings during the first half of a forward traverse being repeated after from 1 to 2 hr. had elapsed. The differences between such repeated measurements were no greater than the differences expected from an error analysis, nor were they greater than the differences between measurements which were repeated immediately. Even though the liquid-holdup determinations represent data taken over rather long periods of time, small, brief variations in the steady state operations appeared to have no significant effect.

VOID-FRACTION MEASUREMENTS

Measurements of the void fraction as a function of quality are given in Figure 6. Eddy's measurements (*6*) were made with horizontal flow at a pressure of 25 lb./sq. in. abs. The Martinelli correlation, established for horizontal flow at atmospheric pressure with no mass transfer, is low by about 10%. The data of Schwarz for horizontal and vertical flow at 80 atm. (1,180 lb./sq. in. abs.) as well as the Martinelli curve for 1,500 lb./sq. in. abs. are also included.

Figure 7 offers a comparison of the correlations and predictions for R_l vs. q and illustrates the wide discrepancies existing in the current literature on two-phase flow. The homogeneous model predicts values for R_l which owing to the assumption that the average linear velocities in each phase are equal, are about 80% lower than the experimental curve. Levy, who considered horizontal two-phase flow with no mass transfer (*10*), predicted R_l values considerably higher than Martinelli's curve. Dengler experimentally determined R_l for vertical steam-water flow with heat transfer to the mixture from the tube wall (*5*). Although his values are also higher than Martinelli's, the curves actually cross, and better agreement with the experimental curve might be expected at higher qualities. Wissler (*18*) independently

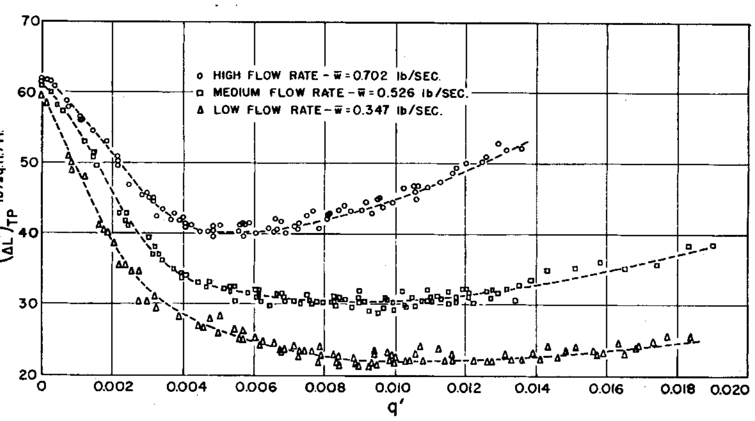

Fig. 10. Static-pressure-drop data for all measuring stations.

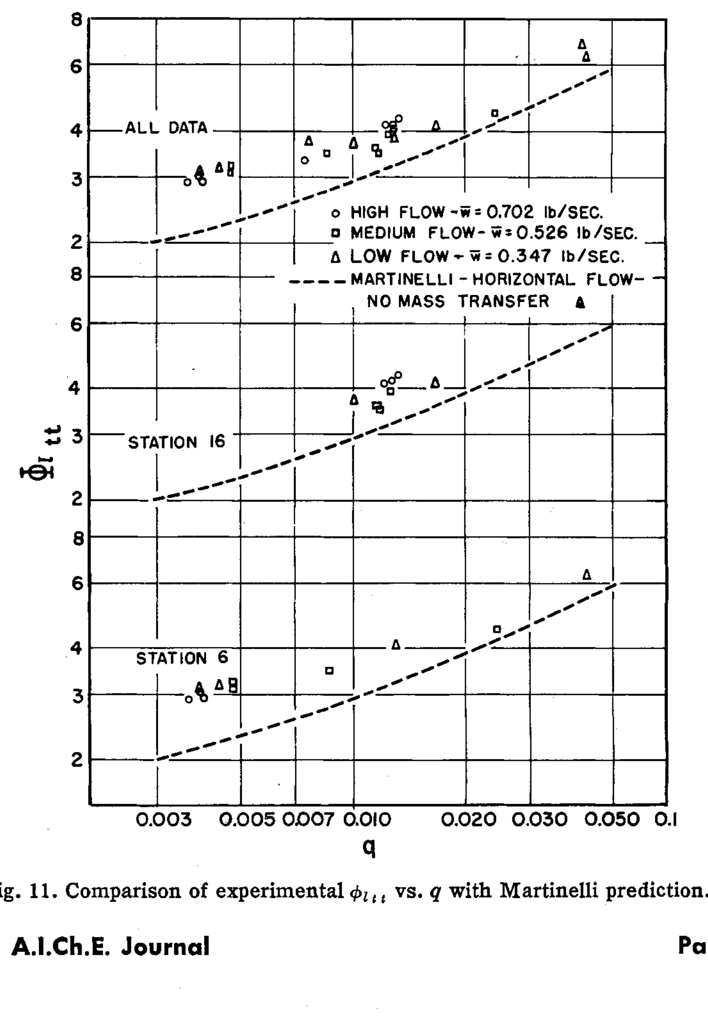

Fig. 11. Comparison of experimental ϕ_{ltt} vs. q with Martinelli prediction.

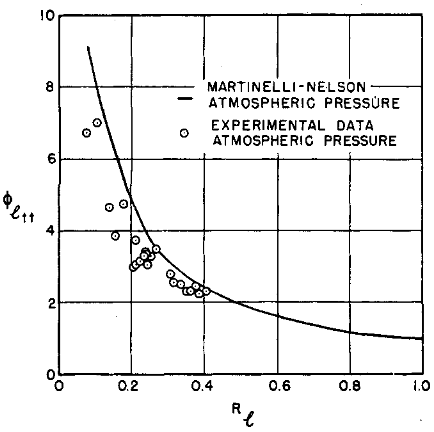

Fig. 12. Plot of ϕ_{ltt} vs. R_l.

utilized a series of two-phase runs in the natural-circulation loop described in this article to evaluate a system of constants for three empirical relationships, one of which essentially defines R_l as a function of the pressure and specific enthalpy of the mixture. His correlation fits the average experimental values quite well (usually within 10% or better) over the range of quality studied.

Detailed examinations of the Martinelli values for R_g reveal that at high pressures and low qualities the Martinelli curves yield values greater than those calculated by the homogeneous method. Use of these Martinelli values leads to predicted momentum pressure drops which exceed the values calculated by the homogeneous method. One would expect that the homogeneous method would set an upper limit for R_g.

Even though the scatter of the data for R_g vs. q is within predicted error limits, further analyses were made in an attempt to discover regular patterns for the scatter. Figure 8 shows the data characterized by flow rate and station number. Neither the flow rate nor the position appear to have significant effects on the relationship between R_g and q.

The attempts to interpret the gamma-absorption data in terms of detailed liquid distribution were not successful (*16*); however, other less exact relationships are of interest. Figure 9 shows the chordal lengths of steam found at the center of the tube plotted against the corresponding values for R_g. The broken line represents the condition for all the water in an annular ring. The relative amount of water existing in the steam core is indicated by the horizontal distance between a point and the line. At station 16 (near top of riser) there is no apparent flow effect, and the assumption that all water is in an annular ring is valid. Further, no flow rate effect was apparent at station 6 (near bottom of riser), but a significant amount of water appears to be in the steam core at the lower values of R_g.

TWO-PHASE PRESSURE DROPS

Table 1 lists the pressure drop per unit length evaluated for the head term $(\Delta P/\Delta L)_H$, the momentum change $(\Delta P/\Delta L)_M$, the frictional two-phase pressure drop $(\Delta P/\Delta L)_{TPF}$, and the static pressure differences $(\Delta P/\Delta L)_{TP}$. Comparisons are made by use of the homogeneous and Martinelli models. The homogeneous model leads to two-phase frictional and momentum pressure drops which are generally higher than those obtained from the Martinelli correlation, whereas the head terms are substantially lower. The homogeneous model yielded values for the total pressure drop per foot which were as much as 60% higher to 45% lower than the experimental values. Experimental values of R_l were used to calculate the head and momentum terms in the Martinelli model. The data given in Table 1 represent conditions at stations corresponding to void-measuring positions. Static-pressure-drop data for all stations are given in Figure 10. The frictional two-phase pressure-drop relations are compared in Figure 11, which is a plot of ϕ_{ltt} vs. q. The experimental values of ϕ_{ltt} differ by as much as 30% from the Martinelli curve. The effects of flow rate and position are not apparent. Use of Martinelli's values for R_l in computing the head and momentum terms would have resulted in a better fit of the ϕ_{ltt} values with the Martinelli curve. Another comparison of the experimental values of ϕ_{ltt} with the Martinelli values is given in Figure 12, which is a plot of ϕ_{ltt} vs. R_l. An approximation of the Martinelli curves (independent of pressure) is $\phi_{ltt}\ R_l \cong 1$.

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NOTATION

c = distance between inner tube walls
D = inside diameter of tube
f = Fanning friction factor
g = acceleration due to gravity
g_c = proportionality factor
G = mass velocity
I = radiation intensity
l = chordal length of water in tube
m = chordal length of steam in tube
P = static pressure
q = calculated quality, lb. steam/lb. mixture
q' = corrected quality
R = fractional area or volume occupied by one phase
v = specific volume
x = distance which gamma beam passes through absorber
z = distance from a horizontal base plane
$\Delta P/\Delta L$ = pressure drop per unit length

Greek Letters

β = linear gamma-absorption coefficient
μ = dynamic viscosity

$$\phi_{ltt} = \left[\left(\frac{\Delta P}{\Delta z}\right)_{TPF} \Big/ \left(\frac{\Delta P}{\Delta z}\right)_{lF}\right]^{1/2}$$

for short increments in z

Subscripts

F = frictional quantity
g = property of vapor
H = a head quantity
l = property of liquid
m = result of attenuation in metal
M = a momentum change quantity
n = pressure tap number
s = property of vapor
TP = properties of two-phase mixture
w = property of liquid

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